JOURNAL OF MATERIALS SCIENCE 27 (1992) 6707–6714

Statistical study of the effect of subcritical crack growth on thermal shock resistance

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An experimental study of the effects of subcritical crack growth on thermal shock damage is presented, based on a statistical analysis of the retained strength distribution. Single-quench thermal shock and thermal shock fatigue tests were performed in a room-temperature distilled water bath on glass microscope slides. Experimental results indicate that subcritical crack growth effects are observable in the shock testing of glass slides in terms of systematic shifts in the retained strength distribution.

1. Introduction

A thermal quench may cause the pre-existing cracks in ceramic components to propagate, which degrades the fracture strength of the shocked components relative to that of the non-shocked components [1]. Badaliance *et al.* [2] proposed that pre-existing cracks could propagate in a single thermal quench below the critical quench temperature difference, ΔT_c, and that subcritical crack growth had a significant effect on the thermal shock resistance of soda-lime glass. In contrast to Badaliance *et al*'s results, Ashizuka *et al.* [3] presented experimental results indicating that subcritical crack growth effects were insignificant to the thermal shock resistance of borosilicate glass. As will be discusssed in the following sections, the difference between Badaliance *et al*'s results [2] and those of Ashizuka *et al.* [3] may be related more to their techniques for assessing subcritical crack growth, rather than differences between the tested materials (soda-lime glass and borosilicate glass).

In this study subcritical crack growth was investigated by comparing the fracture strength distribution of annealed glass to that of quenched (thermally shocked) glass. In addition to single-quench thermal shock testing, subcritical crack growth was evaluated in terms of thermal shock fatigue damage (cyclic thermal shock).

2. Experimental procedure

The specimens used in this experiment were commercial glass microscope slides of 7.6 cm × 2.54 cm × 0.12 cm (VWR Scientific Inc., San Francisco, CA). The as-received slides were annealed in air in an electric furnace at 650 °C for 0.5 h and cooled freely in the furnace to room temperature. Using a commercial testing machine with a crosshead speed of 0.1 cm min^{-1}, the flexural fracture strength of the annealed specimens was determined using a three-point bend fixture with a span of 4.5 cm. The fracture strength, S, was calculated by

$$S = \frac{3PL}{2BD^2} \qquad (1)$$

where P and L are the mechanical load and the span between supports, respectively. B is the slide width and D is the thickness of the glass slides.

During the single-quench thermal shock testing, the annealed slides were set on an alumino-silicate refractory board in an electric furnace initially at room temperature. When the furnace temperature was stable for at least 0.5 h at a predetermined value, the refractory board was quickly removed from the furnace and the slides were dumped into a room-temperature distilled water bath. The quenched slides were removed from the water bath and were allowed to dry naturally in room-temperature air for at least 5 h. The retained fracture strength of the quenched slides was then measured. To achieve statistically reliable estimates of fracture strength, thirty specimens were fractured for each quench temperature difference, ΔT.

In addition to single-quench testing, cyclic thermal shock (fatigue) was performed. An annealed slide was suspended in a specimen holder therefore translated back and forth between a water bath and the hot zone of an electric furnace (Fig. 1). The slide was maintained in the furnace for 45 min and then suddenly dropped into the water bath. The distance from the hot zone of the furnace to the surface of the water bath was about 0.5 m and the specimens traversed this distance in about 0.3 s. During each thermal shock, the specimen remained in the water bath for 7 min; 8 min were required to elevate the specimen slowly and return it to the furnace's hot zone. After each slide was subjected to a pre-determined number of thermal shock cycles, the retained fracture strength of the shocked specimen was measured in three-point bend. At least 13 thermally fatigued glass slides were tested for each ΔT.

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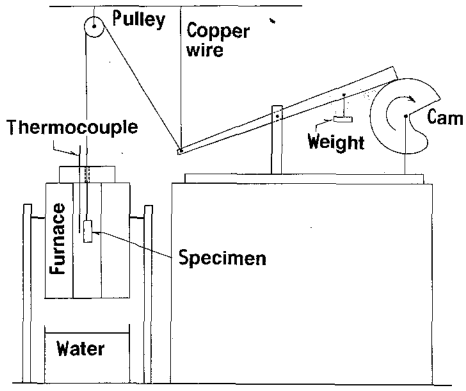

Figure 1 Schematic drawing of the furnace and apparatus for thermal shock fatigue testing.

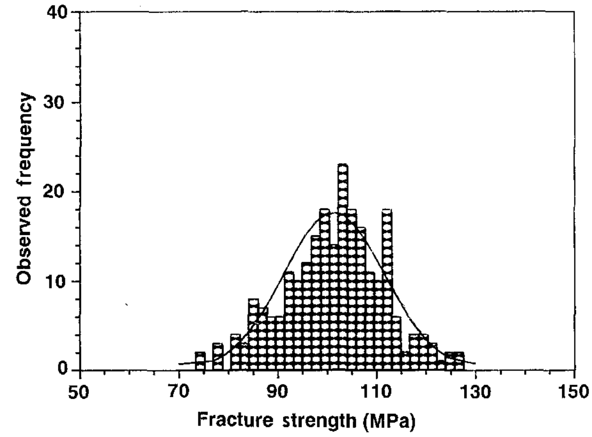

Figure 2 Fracture strength histogram for annealed glass-slide specimens fractured in three-point bend.

3. Results and discussion

3.1. Candidate fracture strength distributions

The strength distribution for a group of monolithic ceramic specimens is related to the distribution of pre-existing critical flaws in the specimens [4, 5]. While the Weibull distribution function is typically used to describe fracture strength distributions [6, 7], distributions other than the Weibull function may provide a better fit to the strength data in some cases [4, 5, 8, 9]. For example, Doremus found that the normal distribution function fit the static strength data for Pyrex glass better than the Weibull distribution [4]. Shimokawa and Hamaguchi [8] reported that the log normal function fit the fatigue data for carbon fibre/epoxy matrix composite specimens better than did the Weibull function. In this study, the fracture strength histogram for 239 annealed glass-slide specimens is approximately symmetrical, so that the normal distribution function is one of the natural candidates to describe the data (Fig. 2). Consequently, in this study the three candidate distribution functions used to analyse the distributions of the fracture strength for the annealed glass slides were the two-parameter Weibull, the normal, and the log normal functions.

Applying the maximum likelihood method [10–12] to the annealed glass-slide fracture strength data yielded the statistical parameters for the normal, log normal, and Weibull distribution functions (Table I). To measure the discrepancy between the fracture strength data and the three candidate distribution functions, a goodness-of-fit test is required. The goodness of fit test employed in this study is based on the empirical distribution function (EDF) and the Kolmogorov–Smirnov test [13, 14]. The EDF, $F_n(y)$, is defined as [14]

$$F_n(y) = \frac{\text{number of observation} \leqslant y}{n} \qquad -\infty < y < \infty \tag{2a}$$

$$F_n(y) = \frac{i}{n} \qquad y_i \leqslant y < y_{i+1} \tag{2b}$$

$$F_n(y) = 0 \qquad y < y_1 \tag{2c}$$

$$F_n(y) = 1 \qquad y_n \leqslant y \tag{2d}$$

$y_1 < y_2 < y_3, \ldots, < y_n$ are the order statistics for the fracture strength data, y_i, for a random sample of size n. $F_n(y)$ is the step function illustrated in Fig. 3. The three continuous curves in Fig. 3 represent the cumulative distribution functions (CDF) of the normal, log normal, and Weibull distributions with the parameters listed in Table I. In the Kolmogorov–Smirnov test, a good fit requires that D_n be small, where D_n is defined in terms of the greatest vertical difference between the CDF and the EDF [13], such that

$$D_n = \max(D_n^+, D_n^-) \tag{3a}$$

TABLE I Parameters for the normal, log normal, and Weibull distribution functions, as calculated from maximum likelihood estimators.

Normal:	mean	$\hat{\mu} = 101.38$ MPa
	variance	$\hat{\sigma}^2 = 104.90$ MPa^2
Log normal:	mean	$\hat{\mu} = 4.6137$
	variance	$\hat{\sigma}^2 = 1.0619 \times 10^{-2}$
Weibull[a]:	expected value	$E(y) = 101.17$ MPa
	variance	$\text{Var}(y) = 127.33$ MPa^2
	Weibull parameters	$\hat{b} = 106.00$ MPa
		$\hat{m} = 10.834$

[a] The cumulative distribution function for the two-parameter Weibull distribution is $F(y) = 1 - \exp[-(y/b)^m]$.

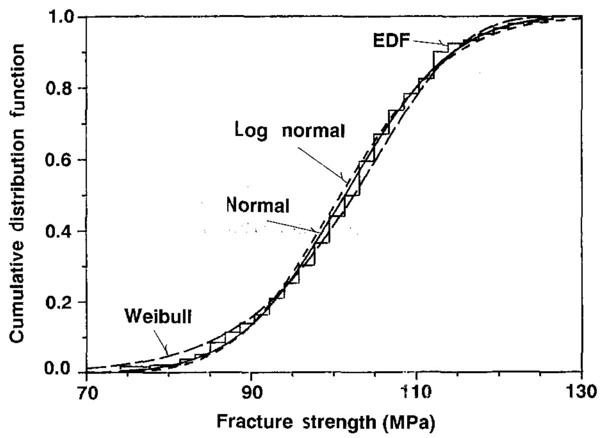

Figure 3 Illustration of cumulative distribution functions and the empirical distribution function for the fracture strength data of annealed glass slides.

TABLE II Statistics for the empirical fracture strength distribution function obtained from the goodness-of-fit test (Kolmogorov–Smirnov test). The statistics D_n^+, D_n^-, and D_n are defined by Equations 3a–c

	D_n^+	D_n^-	D_n
Normal	0.0478	0.0685	0.0685
Log normal	0.0521	0.0867	0.0867
Weibull	0.0787	0.0374	0.0787

$$D_n^+ = \max_{1 \leqslant i \leqslant n} [F_n(y_i) - F(y_i)] = \max_{1 \leqslant i \leqslant n} \left[\frac{i}{n} - F(y_i)\right] \quad (3b)$$

$$D_n^- = \max_{1 \leqslant i \leqslant n} \left[F(y_i) - \frac{i-1}{n}\right] \quad (3c)$$

The Kolmogorov–Smirnov statistic (for which the statistical parameters are estimated by maximum likelihood estimators) shows that the normal distribution corresponds to the smallest D_n. However, because the critical value in the Kolmogorov–Smirnov test (two-sided test at significance level $\alpha = 0.05$) is 0.0878, the normal, log normal, and Weibull distributions all fit this study's strength data for annealed glass slides about equally well (Table II). For convenience, the normal distribution is used for the thermal shock resistance analysis in the next section.

3.2. Subcritical crack growth

The literature does not agree on the significance of subcritical crack growth on thermal shock resistance in ceramics [1–3]. Therefore, it is appropriate here to review briefly the thermal shock literature as it applies to subcritical crack growth. In his 1969 model of thermal shock damage in ceramics [1], Hasselman proposed that: (1) pre-existing cracks propagate when quenched above a critical quench temperature difference, ΔT_c, (2) cracks are stable (do not grow) below ΔT_c, and (3) dynamic crack growth causes a discontinuous drop in the retained fracture strength at ΔT_c.

The effects of subcritical crack growth on thermal shock, which were not included in Hasselman's 1969 theory [1], are treated for single-quench thermal shock in a 1974 paper by Badaliance *et al.* [2]. Subcritical crack growth was modelled by taking into account the propagation of pre-existing cracks for quench temperature differences below ΔT_c. The numerical calculation yielded the critical quench temperature difference $\Delta T_c = 147\,°C$. When subcritical crack growth was ignored, a ΔT_c of 238 °C was obtained. Badaliance *et al.* inferred that the large discrepancy (91 °C) was due to thermal shock-induced subcritical crack growth. In modelling the ΔT_c change due to subcritical crack growth, Badaliance *et al.* used $K_o = 0.248\,\mathrm{MPa\,m^{1/2}}$ and $K_c = 0.749\,\mathrm{MPa\,m^{1/2}}$, where K_c is the critical stress intensity and K_o is the threshold for subcritical crack growth. The values of K_o and K_c adopted by Badaliance *et al.* result from static fatigue testing under a mechanical loading [15].

In another key study of the effect of subcritical crack growth on thermal shock behaviour, Ashizuka *et al.* [3] quenched heated borosilicate glass rods into a room-temperature water bath. The retained strength of the shocked borosilicate glass rods was measured in a liquid nitrogen bath and in a room-temperature water bath. It was assumed that the moisture-free environment of the liquid nitrogen bath would provide "baseline" values of retained fracture strength, free from subcritical crack growth effects. Ashizuka *et al.* [3] calculated K_1 associated with the appropriate ΔT from

$$K_1 = YC^{1/2}\frac{\Delta T\alpha E}{(1-\nu)}F(B) \quad (4a)$$

$$K_c = \sigma YC^{1/2} \quad (4b)$$

where K_1, E, α, ν, C, and Y are the stress intensity factor, elastic modulus, thermal expansion, Poisson's ratio, flaw size, and geometric constant, respectively. The $F(B)$ is a function of Biot's modulus, B, [2]. To determine the inert strength, σ, both the bend fixture and the specimens were immersed in a liquid nitrogen bath. The specimens were subsequently fractured in four-point bend at a crosshead speed of 0.5 mm min^{-1} and an approximate stressing rate of 93.3 MPa min^{-1} [3]. Using Equations 4a and b, Ashizuka *et al.* [3] inferred that $K_o \approx 0.9\,K_c$ and thus subcritical crack growth should be minor.

The differences in technique between the studies of Badaliance *et al.* and Ashizuka *et al.* were that (1) Badaliance *et al.* utilized static fatigue data to theoretically evaluate the effects of quench-induced subcritical crack growth, and (2) Ashizuka *et al.* experimentally measured the retained fracture strength for thermally shocked specimens and then statistically inferred the effects of subcritical crack growth. Badaliance *et al.*'s approach was thus theoretical while Ashizuka *et al.*'s approach was experimental.

3.2.1. Evolution of fracture strength degradation

In our study, the fracture strength of annealed glass slides was characterized by a normal distribution. The evolution of the strength distribution for a group of annealed glass slides depended, for example, on whether or not subcritical crack growth was included in the thermal shock damage process. We discuss the evolution of the crack damage (a) neglecting subcritical crack growth, and (b) including subcritical crack growth.

3.2.1.1. No subcritical crack growth effect. In 1983, Lewis [16] proposed that because the critical flaw sizes of a group of brittle components were characterized by a distribution, the mean retained strength of quenched components should gradually decrease as ΔT increased. Lewis's concept [16] of the evolution of the fracture strength distribution (suggested in 1955 by Manson [7]) is experimentally tested in this paper

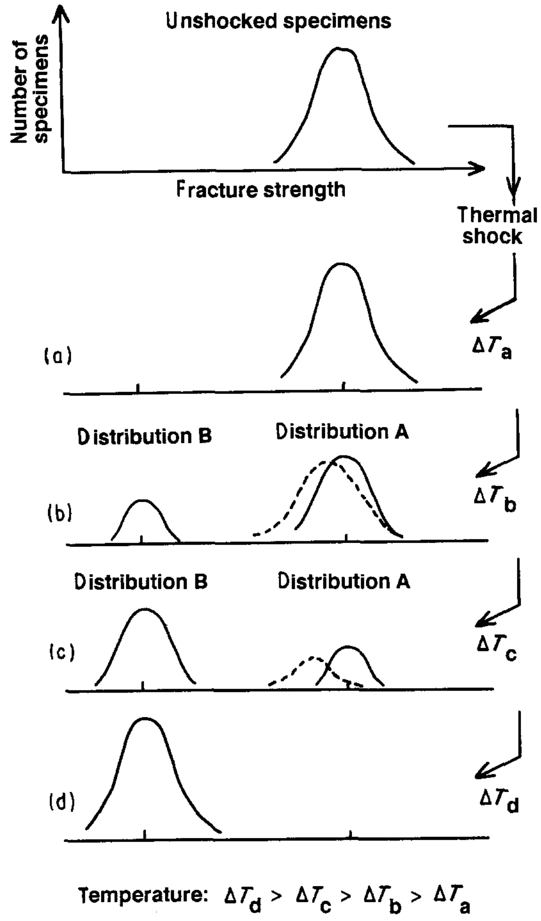

Figure 4 Schematic representation of the evaluation of fracture strength degradation.

using a larger number of glass microscope slides (239 slides were fractured to determine the as-annealed strength distribution and a total of 180 slides were fractured in the single-quench tests).

If transient thermal stresses are sufficiently mild ($K_{thermal} < K_c$ for each specimen), then no strength degradation occurs and the initial fracture strength distribution is not altered (Fig. 4a). When thermal stresses are extremely severe ($K_{thermal} > K_c$ for every slide in the total population of glass slides), the strength of each specimen drops as the critical flaws in the specimens extend (Fig. 4d). When the shock severity is intermediate ($K_{thermal} > K_c$ for some fraction of the slide population), the retained strength distribution breaks into two clusters and becomes bimodal (Fig. 4b and c) [16].

In this study, the cluster with lower fracture strength is defined as distribution B (Fig. 4b and c). The cluster with higher fracture strength is defined as distribution A. The strength will drop for that fraction of specimens for which $K_{thermal} > K_c$ because the critical flaws extend during thermal shock, while (neglecting subcritical crack growth) the strength remains unchanged for those specimens where $K_{thermal} < K_c$.

3.2.1.2. Subcritical crack growth effect included. If subcritical crack growth occurs during thermal shock, the evolution of retained fracture strength will differ from that suggested above, in that there will be additional crack growth regimes and an additional crack growth criterion.

If $K_{thermal} > K_c$, the critical flaws are subjected to "pop-in" crack growth at the initial stage of the thermal shock process. Thus, no subcritical crack growth effect on strength degradation is expected (Fig. 4d). If $K_{thermal} < K_o$ for all slides, then no crack extension will occur by either subcritical or pop-in growth (Fig. 4a).

If $K_c > K_{thermal} > K_o$, then critical flaws will not experience pop-in growth, but they can be extended subcritically. Because subcritical crack growth typically occurs at a relatively low velocity as compared to a pop-in type crack growth, we would expect subcritical crack growth to produce shifts in the mean strength of distribution A (dashed line in Fig. 4b and c), as opposed to the drastic transformations in strength possible in pop-in crack growth.

In order to test for systematic shifts in the retained strength distribution as a function of ΔT, we must first approximate the form of the initial strength distributions. Histograms of the retained fracture strength data (distribution A) for thermally shocked specimens are then compared to the normal distribution determined in Section 3.1 (Fig. 2) for the annealed specimens.

3.2.2. Experimental results of fracture strength degradation for single-quench thermal shock

The glass microscope slides in a portion of this study were subjected to a single quench into a room-temperature water bath. For specimens shocked at a quench temperature difference of 150 °C, the retained fracture strength distribution (solid curve in Fig. 5a) shifts slightly to the left when compared to the strength distribution of the annealed glass slides (dashed line in Fig. 5a) (see Appendix 1). This small shift to the left (toward lower strengths) in the retained strength distribution is interpreted by the authors as indicative of the onset of strength degradation. The histograms of the retained fracture strength for glass slides shocked at $\Delta T = 160$, 170, 180 and 190 °C show the development of a bimodal distribution (Fig. 5b–e). Distribution B (which is similar to the distribution shown schematically in Fig. 4b and c), represents the retained fracture strength of slides quenched severely enough to cause pop-in crack growth. In addition, distribution A shifts progressively to the left toward lower strength values, as compared with the strength distribution of annealed glass slides (dashed line). The experimental strength data in Fig. 5b–e breaks into two clusters. The cluster with higher fracture strength is distribution A. The detailed procedure by which distributions of type A were determined is given in Appendix 1.

Distribution A disappears in the retained strength data for a ΔT of 200 °C (Fig. 5f). The absence of a distribution of type A suggests that each specimen experienced pop-in crack growth. Thus the entire strength distribution was converted into a distribution of type B, which shows a single mode located at relatively low strength values.

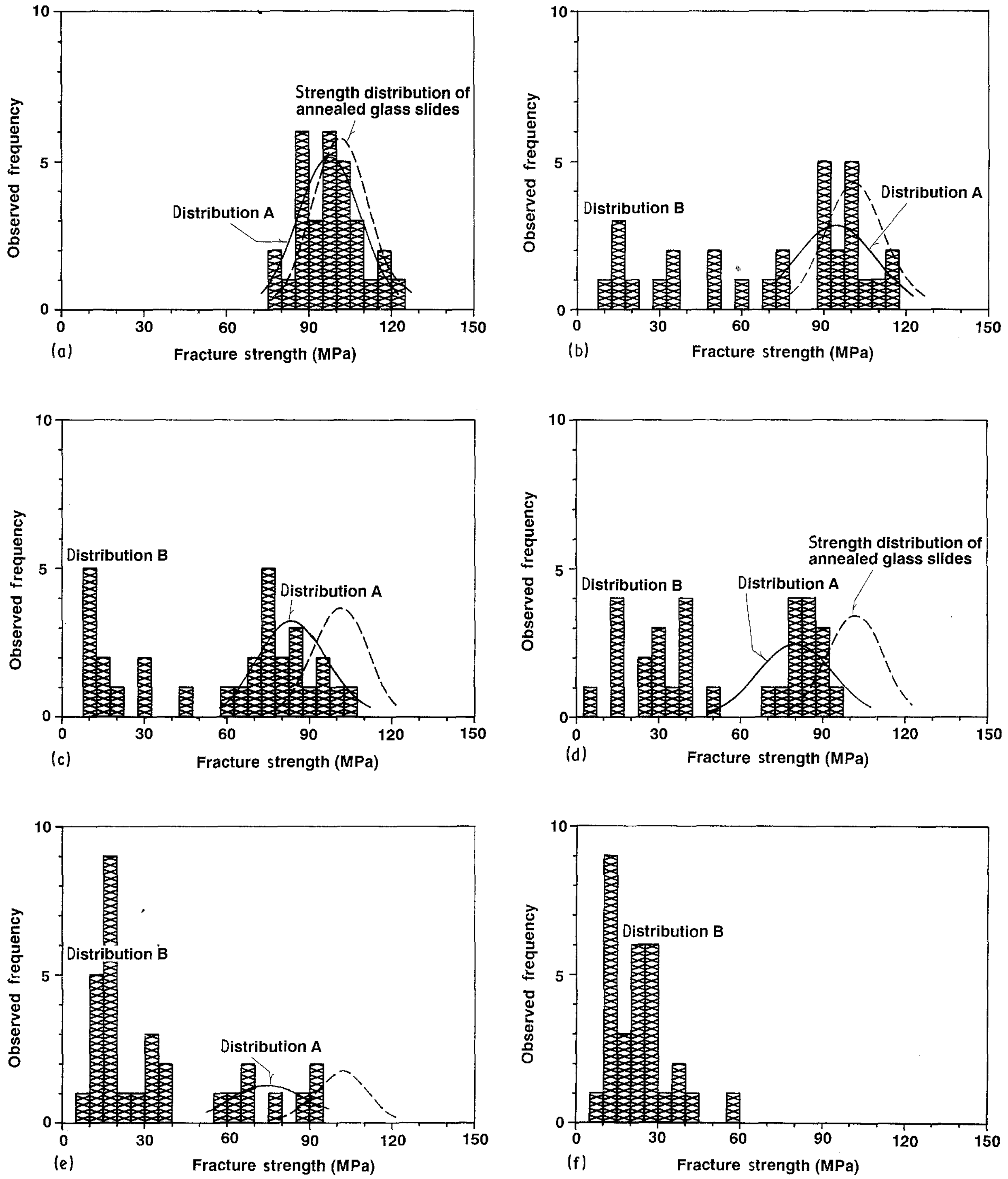

Figure 5 Retained fracture strength of glass slides following a single quench into a room-temperature water bath at $\Delta T =$ (a) 150 °C, (b) 160 °C, (c) 170 °C, (d) 180 °C, (e) 190 °C, (f) 200 °C. (---) The fracture strength distribution of annealed glass slides.

The strength shift for distribution A was attributed to subcritical crack growth. Comparison of the mean strength of distribution A (Fig. 5a–e) and the mean strength of annealed glass slides indicates that the strength shift, $\Delta\mu$, increases monotonically from 4.28 MPa for a ΔT of 150 °C to 26.56 MPa for a ΔT of 190 °C (Table III).

The apparent temperature effect on subcritical crack growth agrees qualitatively with static crack propagation results for silica reported by Sakaguchi *et al.* [17] and dynamic fatigue results by Ritter *et al.* [18]. Sakaguchi *et al.* tested compact tension specimens of fused quartz under static tensile stress in distilled water. Ritter *et al.* measured the dynamic fatigue of indented soda-lime glass in distilled water using a ring-on-ring test fixture. In both studies the subcritical crack-growth rate increased with increasing water temperature [17, 18].

To compensate for the effects of subcritical crack growth, the retained strength data at each ΔT between 150 and 190 °C (Fig. 5a–e) were shifted by $\Delta\mu$ (see Table III and Fig. 6). This shift in strength corresponds to a shift in critical quench temperature difference from the actual quench data $\Delta T_c \approx 175$ °C to a "shifted" value $\Delta T_c \approx 190$ °C. In this paper, the ΔT corresponding to the 50% probability level of failure (see Fig. 5c and d) was considered to be the critical quench temperature difference, ΔT_c. The shift in ΔT_c

TABLE III Thermal shock induced changes in the mean strength, μ, and the standard deviation, σ, for the retained strength distribution as a function of the quench temperature difference, ΔT. The difference in mean strengths, $\Delta\mu$, measures the extent of slow (subcritical) crack growth (units of strength: MPa)

ΔT (°C)	Total shocked specimens		Part A of bimodal distribution		Slow crack growth effects	New distribution without slow crack growth
	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\mu}_A$	$\hat{\sigma}_A$	$\Delta\hat{\mu} = 101.38 - \hat{\mu}_A$	$= \hat{\mu} + \Delta\hat{\mu}$
0	101.38	10.2				
150	97.41	15.3	97.41	11.5	4.28	101.38
160	73.3	34.1	94.25	14.7	7.13	80.43
170	60.75	32.1	83.59	12.1	17.79	78.54
180	58.1	28.8	80.13	13.7	21.25	79.35
190	35	26	74.82	13.7	26.56	61.56
200	22.5	10.7	0		[a]	

[a] Because all specimens for $\Delta T = 200$ °C were subject to "pop-in" crack growth, the subcritical crack growth effect could not be evaluated.

attributable to subcritical crack growth was much less than the subcritical crack growth-induced shift (about 91 °C) that Badaliance *et al.* [2] inferred from their data and computations.

The retained strength evolution for the glass slides shocked in this study indicates that subcritical crack growth does play a role in the thermal shock damage process.

3.2.3. Fracture strength degradation during cyclic thermal shock

In addition to single-quench testing, cyclic thermal shock of the glass slides was analysed in terms of subcritical crack growth. For single quench testing, the retained fracture strength began to decrease at a ΔT of about 150 °C, with an increase in the magnitude of the error bars (Appendix 2) for the strength degradation curve at about 160 °C (Fig. 6). Under cyclic thermal shock conditions, thermal shock damage appeared at temperatures below 150 °C. The magnitude of the thermal-shock induced strength drop also increased as the number of thermal shock cycles increased (Fig. 7a).

To make Fig. 7b more readable, the error bars have been omitted. However, the magnitude of the error bars for Fig. 7b are shown in the corresponding data points in Figs 6 and 7a. The presence of thermal fatigue effects implies that pre-existing cracks can extend during each quench cycle, although the growth tends to saturate for the lower ΔT values. For example, the strength of the slides quenched at $\Delta T = 130$ and 140 °C tend toward a saturated damage level for the number of thermal cycles performed in this study, while the strength drops off precipitously

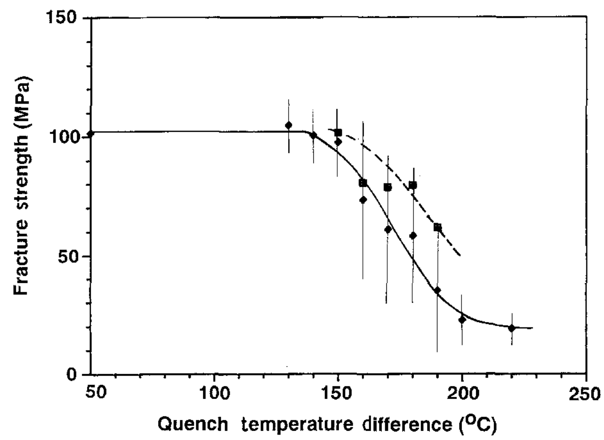

Figure 6 A plot of retained fracture strength versus ΔT. (——) The original strength data curve (single thermal shock), which becomes the dashed line (---) after compensating for subcritical crack growth effects (see Table III).

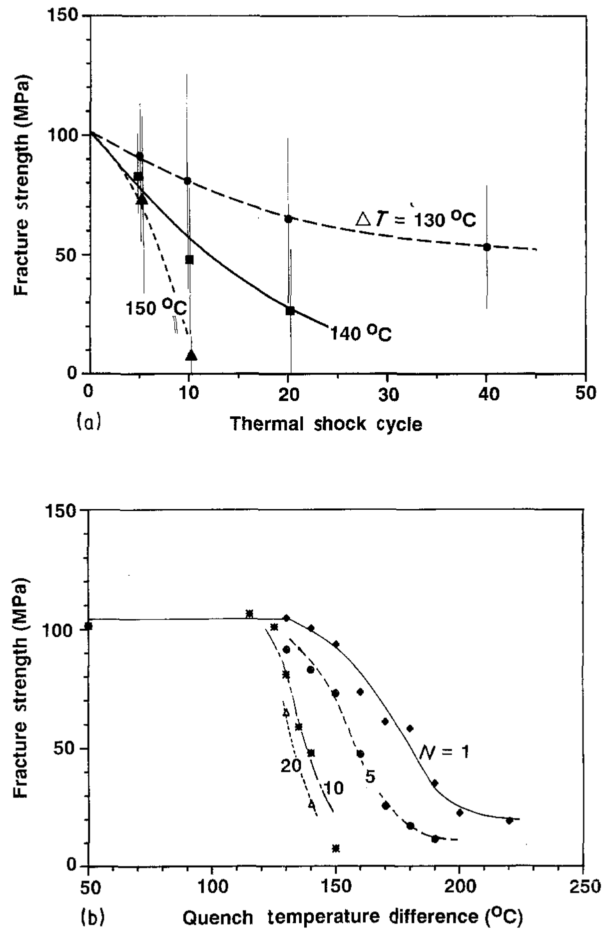

Figure 7 (a) Influence of a cumulative number of thermal shock cycles on the retained fracture strength of the glass slides repeatedly shocked below ΔT_c; where ΔT_c is the critical quench temperature difference determined from single-quench testing (Fig. 5). (b) Variation of retained fracture strength with respect to ΔT and the cumulative number of thermal shock cycles, N.

for specimens shocked repeatedly at a ΔT of 150 °C. Therefore, Fig. 7a and b demonstrate that subcritical crack growth can occur below the critical quench temperature difference (which corresponds to a stress intensity factor below K_c).

Subcritical crack growth is a complex function of temperature and chemical environment. In addition, subcritical crack growth depends on K_o and K_c. For example, as K_c increases, pop-in crack growth decreases. Also, thermal stresses are very strong functions of time and position, thus these stresses are even more difficult to characterize than stresses in quasi-static loading experiments dealing with subcritical crack growth [17–22]. The relatively small shift in ΔT_c attributable to subcritical crack growth agrees qualitatively with Ashizuka *et al.*'s inference [3] that subcritical crack growth was insignificant to the thermal shock resistance of borosilicate glass rods.

4. Conclusion

As ΔT varies in thermal shock experiments on brittle materials, shifts in the retained fracture strength distributions are indicative of the relative contributions of subcritical (slow) crack growth (Figs 4 and 5). The initial step in the experimental assessment of subcritical crack growth in thermal shock is to determine the strength distribution of a population of unshocked specimens. In this study, a Kolmogorov–Smirnov goodness-of-fit statistic indicated that the normal, log normal, and Weibull distribution functions fit the fracture strength for unshocked annealed slides about equally well. For convenience, a normal distribution was used for the thermal shock resistance analysis.

The strength degradation observed for thermal shock fatigue tests qualitatively indicates that slow crack growth (subcritical crack growth) occurs below ΔT_c (below K_c). A non-negligible subcritical crack growth contribution to thermal shock damage was also demonstrated experimentally via a statistical analysis of the retained fracture strength data for single-quenched glass slides. However, subcritical crack growth decreased ΔT_c by about 15 °C which is much less than the 91 °C shift in ΔT_c reported by Badaliance *et al.* [2].

Acknowledgement

The authors thank Dr James Stapleton, Department of Statistics, Michigan State University, for helpful discussions.

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Received 2 September 1991
and accepted 7 May 1992

Appendix 1. Determination of distribution A

For certain ranges of quench temperature difference, ΔT, the schematic drawings in Fig. 4c and d depict the retained strength distribution as separating into two clusters. In Fig. 4c and d, the two clusters (also labelled distributions A and B) are clearly distinguishable. However, in practice the distributions may not exhibit such a clear separation (Fig. 5b). This Appendix proposes a systematic way of approaching this problem.

In order to proceed with the analysis, we assumed that: (1) the shocked specimens without pop-in crack growth (distribution A) exhibit a normal strength distribution similar to that of the annealed specimens, and (2) subcritical crack growth only shifts distribution A, without changing its shape. Consequently, we describe distribution A by a normal distribution with the same standard deviation as the annealed specimens.

In this paper, we determined distribution A as follows.

1. The retained strength data for n specimens were ranked in ascending order such that $y_1 < y_2 < y_3 \ldots < y_{n-1} < y_n$.
2. Distribution A is a subset of the retained strength data. The subset consists of the ordered strength data for $(n - j + 1)$ specimens, which is $y_j, y_{j+1}, y_{j+2}, \ldots, y_{n-1}, y_n$. The jth strength values are selected such that the standard deviation of the subset, $\hat{\sigma}_A$, is approximately equal to σ, the standard deviation of the strength distribution for the annealed glass slides. In this study, σ for the annealed slide glass population was 10.2 MPa.

3. After determining the strength values to include in distribution A, the mean strength, $\hat{\mu}_A$, is then calculated.

In Fig. 5, the solid curves represent the normal distributions with mean μ_A and standard deviation σ_A for the $(n - j + 1)$ specimens that underwent subcritical crack growth. The dashed curves in Fig. 5 illustrate the normal distribution for the $(n - j + 1)$ specimens, but with the mean and standard deviation of the unshocked annealed glass slides ($\mu = 101.38$ MPa and $\sigma = 10.2$ MPa). The dashed curve thus presents the distribution as it would have been without the shift in the strength distribution produced by subcritical crack growth in the glass slides.

Appendix 2. The effect of subcritical and pop-in type crack growth on the magnitude of the error bars for the retained strength of thermally shocked specimens

The fracture strength distribution of annealed glass slides in this study had a standard deviation $\sigma = 10.2$ MPa (Fig. 2). Thermal shock damage caused the strength distribution of shocked slides to form two clusters, with one cluster corresponding to slides that underwent pop-in growth and the other cluster corresponding to slides that underwent slow crack growth only (Fig. 5b–e). Thus, when ΔT is large enough for the strength distribution to become bimodal, the standard deviation of shocked slides then becomes large in comparison with that of annealed slides. The bimodal nature of thermal shock damage (in terms of pop-in and subcritical crack growth) was discussed in an earlier study on the thermal shock behaviour of borosilicate glass [3].

As an example, consider that only five specimens had been thermally shocked at a given ΔT and that three specimens underwent pop-in type crack growth and that the other two specimens experienced subcritical crack growth only. The error bar, which represents two standard deviations in the strength values, would be considerably larger in this case than the corresponding error bars for the as-annealed strength distribution or for the case where all specimens undergo only pop-in growth or only subcritical crack growth.